Financial Crisis and Permanent Output Drop: a Model of Reverse Causality

Nadia Pozdnyakova

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Abstract

Why financial crises are followed by slow recoveries? This paper introduces a channel that generates credit tightening in response to an exogenous trend shock to output, reverting the direction of causality prevalent in the literature. When a negative trend shock hits the economy, endogenous borrowing constraint tightens much more than after a transitory shock. In a heterogeneous household model with a no-default borrowing limit a trend shock causes a sizable fall in credit supply and household debt deleveraging. In contrast, after a temporary shock, household debt increases. The model offers an alternative approach to generating the connection between financial tightening and persistent output losses.

1 Introduction

One feature of financial crises attracting much attention in the literature is the slow recoveries following them. Cerra and Saxena (2008) use panel data on a large set of countries to show that the output losses after a crisis are very persistent and are better described as changes in the trend growth. Haltmaier (2013) and Reifschneider et al. (2015) support the finding by estimating potential output growth and documenting its decline after a financial crisis. Reinhart and Rogoff (2009) also show the prolonged impact of a crisis both on financial (equity and housing prices) and real (output and unemployment) variables.

The existing explanations for the association between financial crises and trend output losses typically suggest that a financial crisis causes the output loss. Apart from the persistent effects that any recession can generate (such as slower capital accumulation and human capital losses due to high unemployment), there are several sources of persistence specific to financial crises. For example, Queralto (2020) suggests that financial tightening restricts the access to innovation funding, resulting in persistently lower productivity, with Ridder (2017) providing empirical support. Ates and Saffie (2016) attribute the fall in productivity to credit tightness preventing the entry of new firms and slowing down the expansion of existing firms.

In this paper, I ask whether there can be a mechanism working in the opposite direction. Can a trend output drop generate financial tightening? The paper offers a new channel connecting permanent output losses with credit crunch through the forward-looking endogenous borrowing constraint, which tightens strongly in response to a trend shock to income but reacts much weaker to transitory recessions. The debt deleveraging occurs after permanent shocks but not after temporary ones.

The approach suggested in the paper is based on the well-documented association between the financial crisis and the credit cycle. For instance, Sufi and Taylor (2021) survey evidence that financial crises tend to be preceded by expansions in private, especially household, credit and are accompanied by debt deleveraging. Moreover, Jordà et al. (2017) show that recessions associated with deleverage or low credit growth tend to last longer. This paper focuses on household debt and models financial tightening as a credit crunch driven by a sharp reduction in household credit supply.

I study an unexpected output contraction in an Aiyagari-type heterogeneous agent model with endogenous no-default borrowing constraint. In this model, a household has an option to default on its liabilities and enter the autarky state, losing access to financial markets forever. The borrowing constraint is such that the household chooses to repay the debt even under the lowest income realization given the expected aggregate state.

In such a setting, highly persistent, less insurable shocks decrease the benefits of access to credit. The incentive to repay the debt is low when the financial market cannot help to smooth consumption. Therefore, the no-default constraint tightens. As a result, after a trend income shock, a drastic contraction in credit supply generates debt deleverage. In contrast, under temporary shocks, the borrowing limit loosens relative to output despite tightening in absolute terms. That allows the households to increase their debt to smooth consumption.

This mechanism creates a strong connection between the persistence of the output shocks and financial tightening absent under the fixed borrowing limit. The magnitude of the credit crunch in the model is substantial: when feeding in the model the shocks to the disposable income estimated from the Euro crisis for Italy, the debt deleverage in the model exceeds the one observed in the data.

The sharp difference between credit market response to trend and transitory shocks stems from the forward-looking behavior of the borrowing limit and, in particular, its sensitivity to the future path of the financial constraint. When an economy converges to a low steady state, the borrowing limit stays permanently tighter, reducing the value of access to the credit market and amplifying the financial tightening during the transition.

In addition, the paper addresses the policy implications of endogenous borrowing constraints. I study a government that wants to support the consumption of low-income households during the recession and lowers the penalty for default. On the one hand, this measure makes it easier for households to increase current consumption by refusing to repay the debt. On the other hand, the policy exacerbates the tightening of the endogenous constraint, so that the households who decide against defaulting must reduce borrowing and decrease consumption. In the quantitative model, the fraction of the defaulting households, who benefit from the policy, is low, and credit tightening hurts a larger share of agents. In aggregate, consumption also decreases.

The paper relates to two strands of the literature. First, the idea of financial constraints tightening in response to a real shock, amplifying it, and making it more persistent is well-studied in the literature. A classic example is Kiyotaki and Moore (1997), where the collateral constraint depends on the value of the productive land. After a negative real shock, the land prices go down, endogenously tightening the constraint. In the context of households, a close analog is collateral constraints based on the housing value, as Mian et al. (2013). The models of this class imply that more persistent shocks would have a larger impact due to a sharper reduction in the net worth of the agent. However, there are no papers studying qualitative differences in the responses to temporary and trend shocks.

Second, the model is close to the body of literature using the no-default borrowing constraint. Sovereign debt models have been widely using this approach since Eaton and Gersovitz (1981) introduced it. It also appears in several heterogeneous household models. Krueger and Perri (2006) study the relation between income and consumption inequality, while Broer (2014) and de Ferra et al. (2021) use the no-default constraint to explain the connection between inequality and capital flows. The novelty of the paper lies in using no-default constraint to study the response of the credit supply and debt to output shocks.

This paper proceeds as follows. In Section 2, I introduce the no-default borrowing limit and illustrate its main properties in a simple analytical setting. In Section 3, I outline a quantitative model of a small open economy inhabited by heterogeneous households subject to endogenous financial constraint. In Section 4, I show that this model generates debt deleverage in response to a trend shock to output but not a transitory shock. Moreover, I show that the magnitude of the deleverage is large and exceeds the one observed in the data. In Section 5, I focus on the policy implications of the no-default constraint and study whether decreasing the cost of default can support the consumption of low-income households during the recession. Section 6 concludes.

2 No-default Borrowing Constraint: an Analytical Example

In this section, I define the no-default borrowing limit as the highest debt the agent is willing to repay if the alternative is entering the autarky. Then, I illustrate the main properties of this limit with a simple example. The key result of this section is the borrowing limit tightening after a negative shock to income, and more so after a persistent shock. This happens because, under high persistence, financial markets provide fewer opportunities for consumption-smoothing and are valued less, making autarky a more attractive alternative to debt repayment.

2.1 Setting

The agent receives deterministic endowment y_t and maximizes his utility from consumption

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $u(\cdot)$ is an increasing and concave utility function.

The agent is able to issue a non-state-contingent one-period bond with the interest rate r, such that $r(1+\beta) = 1$. In the first period, the agent holds the initial debt $a_0 \leq 0$ and chooses whether to default or repay the debt. After default, the agent enters autarky forever and always consume his endowment, similar to Eaton and Gersovitz (1981). The value of autarky $V_A(\{y\}_0^\infty)$ can be written as

$$V_A(\{y\}_0^\infty) = \mathbb{E}_0 \sum_{k=t}^{\infty} \beta^k u(y_k)$$

, where notation implies that the value depends on the whole path of output.

If the agent chooses to repay the debt, he retains the access to financial market and faces no further borrowing constraints. Consumption is chosen optimally. Due to the condition on the interest rate, consumption is constant and equal to a fraction of permanent income, $c = ra_0 + \sum_{k=0}^{\infty} \left(\frac{1}{1+r}^k\right) y_k$. Then, the value of repayment $V_R(\{y\}_0^\infty, a_0)$ is

$$V_R(y_0, a_0) = \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t u \left[ra_0 + \sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^k y_k \right]$$

Proposition 1. *Define the borrowing limit $\underline{a}_0$ as the highest debt the agent would be willing to repay, or assets $\underline{a}$ s.t.*

$$V_R(\{y\}_0^\infty, \underline{a}) = V_A(\{y\}_0^\infty)$$

The borrowing limit $\underline{a}_0$ exists and is unique.

Proposition 1 holds due to the value of autarky $V_A(\{y\}_0^\infty)$ being independent of the assets, while $V_R(\{y\}_0^\infty, a_0)$ monotonically increasing in assets a_0.

Proposition 2. *Given the future path of output $\{y\}_1^\infty$, low current income $y_0^1 < y_0^2$ results in the tighter borrowing limit $\underline{a}(y_0^1) > \underline{a}(y_0^2)$.*

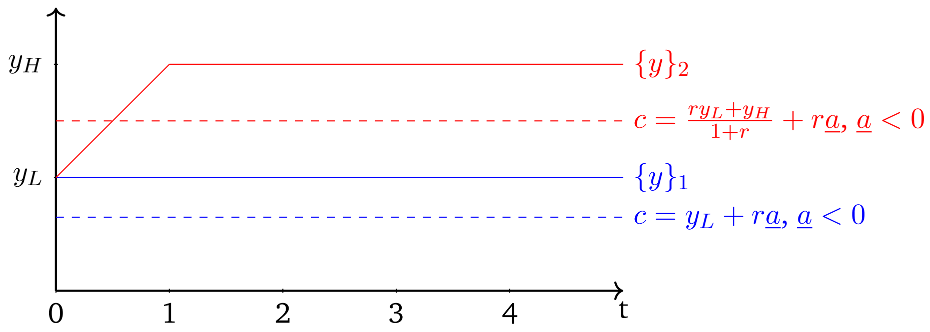
Proof. See Appendix A. □

Intuitively, when the agent is risk-averse, repaying the debt is more costly in low-income states, so the incentive to default is stronger. Therefore, high levels of debt are not sustained in this situation.

Corollary 1. *The result holds if the endowment is stochastic and iid.*

Proof. Both in this case and with the deterministic endowment, the expected value of the future income is independent of the current state, so the proof of Proposition 2 holds without change. □

Figure 1: The Paths of Endowment and Consumption



Notes: The figure illustrates the paths of endowment (solid) and consumption (dotted) after a deterministic fall in endowment lasting for one period or permanently (red or blue correspondingly).

The corollary would be useful in Section 3, helping to determine the type of household determining the constraint in a heterogeneous agent model.

2.2 Persistence and Borrowing Limit

This subsection shows that, given the current endowment, the borrowing limit is tighter for more persistent output shock. I start with illustrating this statement with two extreme examples, permanent and one-period shocks, and then generalize for any AR(1) process.

Permanent drop in endowment. Assume that at period 0, output falls to the value y_L and stays there forever (blue solid line in Figure 1). Denote the path of the output as $\{y\}_1$.

Proposition 3. *In the case of a permanent drop in income, the agent defaults on any debt, so the borrowing limit is zero, $\underline{a} = 0$.*

Proof. Using the restrictions on parameters and the fact that optimal consumption is constant, the condition $V_R(\{y\}_1, \underline{a}) \geq V_A(\{y\}_1)$ can be written as

$$V_R = \frac{1}{1-\beta} u\left(y_L + \frac{1-\beta}{\beta}\underline{a}\right) \geq \frac{1}{1-\beta} u(y_L) = V_D$$

That requires $\underline{a} = 0$. □

Intuitively, the agent faces a trade-off between either repaying the debt and permanently reducing consumption or losing access to the financial market. With a permanent fall in endowment, there are no gains from consumption-smoothing, so the opportunity to borrow has no value. Therefore, the agent prefers to default and enter autarky.

One-period drop in endowment. Assume that at period 0 output falls to the value y_L but at period 1 it restores to $y_H > y_L$ and stays there forever (red solid line in Figure 1). Denote the path of the output as $\{y\}_2$.

Proposition 4. *In the case of a one-period drop in income, there is the borrowing limit $\underline{a} < 0$ such that the agent prefers to repay debt when it is low enough, $a > \underline{a}$.*

Proof. The condition $V_R(\underline{a}, \{y\}_2) \geq V_A(\{y\}_2)$ becomes

$$\frac{1}{1-\beta} u\left(\frac{ry_L + y_H}{1+r} + r\underline{a}\right) \geq u(y_L) + \frac{\beta}{1-\beta} u(y_H)$$

Using the fact that $\beta(1+r)=1$ and multiplying by $1-\beta$, can write it as

$$u\left((1-\beta)y_L+\beta y_H+\frac{1-\beta}{\beta}\underline{a}\right)=(1-\beta)u(y_L)+\beta u(y_H)$$

By the definition of concavity,

$$u\left((1-\beta)y_L+\beta y_H\right)>(1-\beta)u(y_L)+\beta u(y_H)$$

Therefore, $\underline{a}<0$, and the agent will be willing to repay some positive amount of debt. ☐

In this case, the agent gains a positive value from smoothing consumption at period 0. Therefore, the agent would be willing to sacrifice some units of consumption to keep access to the financial markets.

AR(1) drop in endowment. The results can be generalized to the endowment following an AR(1) process, $y_t=y-\rho^t\epsilon$. For any $\rho\in(0,1)$, the agent benefits from smoothing consumption. Thus, there exists a level of debt that would be repaid. However, a more persistent endowment process would generate a tighter borrowing limit.

Proposition 5. *In the case of an AR(1) drop in income, for any $\rho\in(0,1)$, borrowing limit is negative, $\underline{a}<0$. Furthermore, if $\rho_1>\rho_2$, then $\underline{a}_1(\rho_1)>\underline{a}_2(\rho_2)$, so that the borrowing limit tightens with persistence.*

Proof. See Appendix A. ☐

That illustrates the key mechanism of the paper, which will be driving the findings in the quantitative section.

2.3 Borrowing Limit and the Future Constraints

The previous subsection assumed that the agent does not face any financial constraints in the future. However, if the constraints are present and binding, they matter for the determination of the borrowing limit.

To illustrate, take the one-period drop in income and assume that starting from $t=1$, no borrowing is allowed. In that case, after repaying the debt, the agent consumes $y_L+(1+r)\underline{a}$ at $t=1$ and y_H at $t\geq 2$, which is clearly worse than autarky. As future constraints don't allow the agent to smooth consumption, the value of the credit market goes to zero, and the agent would want to default on any debt.

In a more general form, the dependence of the borrowing limit on future constraints is presented in the following Proposition:

Proposition 6. *Suppose that the endowment drops for one period at $t=1$ and the agent faces financial constraint $a\geq a'$. Assume the constraint binds. Then, the no-default borrowing limit $\underline{a}$ is increasing in a', or the borrowing limit tightens for more binding constraint in the future.*

Proof. The condition $V_R(\underline{a},\{y\}_2,a')\geq V_A(\{y\}_2)$ becomes

$$u\left(y_L+(1+r)\underline{a}-a'\right)+\frac{\beta}{1-\beta}u(y_H-ra')\geq u(y_L)+\frac{\beta}{1-\beta}u(y_H)$$

The borrowing limit can be written as

$$(1+r)\underline{a}=u^{-1}\left(u(y_L)+\frac{\beta}{1-\beta}u(y_H)\right)-y_L+a'$$

Want to show that the derivative with respect to a' is positive:

$$\frac{\partial \underline{a}}{\partial a'} = u^{-1\prime}\left(u(y_L) + \frac{\beta}{1-\beta}u(y_H)\right) \cdot u'(y_H - ra') + 1 \geq 0$$

$$\frac{u'(y_H - ra')}{u'\left[u^{-1}\left(u(y_L) + \frac{\beta}{1-\beta}u(y_H)\right)\right]} \geq -1$$

As we know that $u(\cdot)$ is strictly increasing, $u'(\cdot) > 0$. Therefore, the condition holds. $\square$

Despite the willingness of the agent to smooth consumption, financial constraints restrict the opportunities to do so. Thus, for high levels of debt autarky becomes a better alternative than repayment. The borrowing limit is forward-looking and depends not only on the path of income but also on the path of the financial constraint. In Section 4, this result will explain most of the difference between shocks of different persistence.

3 Quantitative Model

In this section, I introduce the model used for quantitative analysis. First, I outline the benchmark heterogeneous household model with incomplete markets based on Aiyagari (1994). Next, I modify the model to include a no-default borrowing limit similar to Eaton and Gersovitz (1981). Finally, I give the details on calibration.

3.1 Benchmark Model

The economy is inhabited by a continuum of infinitely-lived households maximizing their life-time utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}),$$

where c_{it} is consumption of household i at period t, β is a discount factor, $u(c_{it}) = \frac{1}{1-\gamma}c^{1-\gamma}$ is CRRA utility, and γ is risk-aversion parameter.

The households are heterogeneous in income and asset holdings. The income of a household i depends on the aggregate output multiplicatively, $y_{it} = \mu_{it}Y_t$, so that reduction in the aggregate output leads to a proportional fall in idiosyncratic income for all households. The idiosyncratic parameter μ_{it} evolves as an AR(1) process in logs

$$\ln \mu_{it} = \ln \mu_{it-1} + \sigma\epsilon_{it},\ \epsilon_{it} \sim N\left(-\frac{\sigma}{2(1+\rho)}, 1\right),$$

where ρ and σ are persistence and volatility of idiosyncratic income process.

Households have access to incomplete financial markets and are able to save and borrow in a non-state-contingent riskless bond with an interest rate r. The interest rate r is constant and exogenous and can be seen as an international interest rate in a small open economy.

A household is subject to a borrowing constraint

$$a_{it+1} \geq \underline{a},$$

where a_{it} is the asset holdings of a household and $\underline{a}$ is a borrowing limit. In the benchmark model, the limit is exogenous. Otherwise, it is determined endogenously from the no-default condition as described in the following section.

The budget constraint of a household is given by

$$c_{it} + a_{it+1} = y_{it} + (1+r)a_{it}$$

In such a model, the optimal consumption depends on the expected path of income in accordance with the permanent income hypothesis. A permanent fall in output makes the optimal consumption adjust with almost the same magnitude as output[1] and stay at the lower level forever. Access to financial markets would be only marginally helpful in adjusting to this type of shock. At the same type, after a temporary shock, the agents would be willing to dissave or borrow in order to smooth consumption, as their expected lifetime income doesn't change significantly. The benefits of access to the financial market can be high in this case. This distinction will be crucial for the behavior of the borrowing limit described below.

3.2 Endogenous Borrowing Limit

Assume that every period a household with negative assets can choose whether to default or repay the debt. In the case of default, the household pays a fixed utility cost f and looses the access to financial market forever, only consuming its endowment from now on. The value under autarky $V_A(\cdot)$ for a household with contemporary income y_{it} is defined as

$$V_A(y_{it}) = \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t u(y_{it}) - f$$

The value of repayment is given by

$$V_R(y_{it}) = \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t u(c_{it}),$$

where consumption is chosen optimally subject to budget and borrowing constraints.

The financial intermediaries set the borrowing limit $\underline{a}_{it}$ such that to exclude the possibility of a default in the equilibrium. No-default constraint requires that for any agent,

$$V_R(y_{it}, a_{it}) \geq V_A(y_{it})$$

As shown in the previous section, the borrowing limit is tighter for low assets and income[2], so in the heterogeneous agent model, the no-default constraint is the most binding for the agent with the lowest income realization and highest debt. Therefore, the borrowing limit is determined as

$$V_R(\underline{y}_{it}, \underline{a}_{it}) = V_A(\underline{y}_{it}),$$

where $\underline{y}_{it}$ denotes the lowest possible realization of income at time t. As at time $t-1$, when the borrowing decision is made, the realizations of idiosyncratic income shocks for time t are unknown, even the highest-income household has a non-zero probability of receiving the lowest income next period. So, the borrowing limit is uniform for all households independent of their current income.

3.3 Calibration

The period is a year. The coefficient of relative risk-aversion and the annual risk-free rate are set to the standard values in the literature, $\gamma = 2$ and $r = 0.02$. Idiosyncratic income parameters are

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[1] Given the assets are not too large.

[2] Despite the result on income is iid and the idiosyncratic process is AR(1), I guess and verify in the quantitative model that this claim still holds.

Table 1: Model Parameters

Parameter		Value
Risk-aversion	γ	2.00
Risk-free interest rate	r	0.02
Borrowing limit (ss)	$\underline{a}$	-0.225
Persistence of idiosyncratic income	ρ	0.88
Volatility of idiosyncratic income	σ	0.26
Discount factor	β	0.90
Fixed cost of default	f	1.79

Table 2: Targeted and Non-targeted Moments

Variable	Model	Data
Wealth-to-income ratio	0.87	0.87
Hand-to-mouth share	0.23	0.23
Gini index income	0.30	0.30
Income share bottom 75	0.55	0.56
Income share top 10	0.23	0.23
Income share top 5	0.14	0.13
Gini index wealth	0.74	0.74
Wealth share bottom 75	0.11	0.14
Wealth share top 10	0.58	0.65
Wealth share top 5	0.38	0.51

Notes: Hand-to-month share includes households with liquid assets less than two weeks of income. Wealth-to-income ratio is the average ratio of the liquid wealth to annual income. Data moments are taken from Guntin et al. (2020).

taken from Guntin et al. (2020), where they are estimated from Italian micro-level data, with the values $\rho = 0.88$ and $\sigma = 0.26$. The discount factor and the steady-state level of debt are calibrated by targeting the proportion of hand-to-month agents and the average income-to-wealth ratio. The resulting values are $\beta = 0.90$ and $\underline{a} = -0.225$. Another parameter the model needs is the utility cost of default. I calculate it as a value that would generate the debt level $\underline{a} = 0.225$ in the steady state, which gives $f = 1.79$ units of utility, or $f_c = 0.0105$ in terms of consumption for the most constraint agent. The model parameters are presented in Table 1.

The model does reasonably well in matching the non-targeted moments, income and wealth distribution. The values simulated by the model are compared with the data in Table 2.

I study an unanticipated fall in output. Initially, the economy is in a steady state without any uncertainty about the future. After the shock, the path of output is deterministic. In the case of temporary shocks, the output returns to the initial steady-state, while in the case of a trend shock, the output converges to a lower level. The magnitude of the output fall at $t = 1$ is the same for all parametrizations, $\epsilon = 0.05$. The interest rate is exogenous and stays constant at the steady-state level.

In the case of a shock to the trend, the output is expected to evolve as

$$\log Y_t = \log Y_{t-1} + \rho_g^t \epsilon_Y,$$

where ρ_g^t is a parameter governing the speed of convergence to the new steady-state. Following Guntin et al. (2020), who calibrate this parameter to match the aggregate consumption-income elasticity, I set $\rho_g^t = 0.22$.

In the case of a temporary shock, the path of output is

$$\log Y_t = \log Y_{ss} + \rho_Y^t \epsilon_Y,$$

where ρ_Y is the persistence parameter. I study the values $\rho_Y \in [0.80, 0.975]$ to see how the persistence affects the results. For reference, Guntin et al. (2020) use $\rho_Y = 0.9$, which they estimate to be the average persistence of output in the economy.

In the benchmark case, the borrowing constraint stays fixed at the steady-state level throughout the transition. In the endogenous borrowing limit case, the constraint is determined by a comparison between the values of autarky and repayment for the lowest income type. The utility cost of default is assumed to stay constant.

Notice that there is no aggregate uncertainty in the model, so the steady-state no-default constraint is determined under the expectation that the economy won't be hit by any shock next period. After an unanticipated negative shock, the households receive a lower income than expected, so some may prefer to default. In that case, they consume their endowment and never gain access to the financial market again.

4 Quantitative Analysis

This section explores how the no-default borrowing limit generates the household debt deleveraging in response to a trend shock to income. First, I show how the combination of endogenous constraint and a permanent MIT shock causes a sharp tightening of the credit supply, far exceeding the one under a transitory shock and causing a credit crunch. Next, I compare the magnitude of the deleverage with the Italian data during the Euro crisis. I find that the model overestimates the credit crunch. Finally, I present a brief discussion of the model assumptions generating this result and possible strategies to evaluate its empirical relevance.

4.1 Unanticipated shock

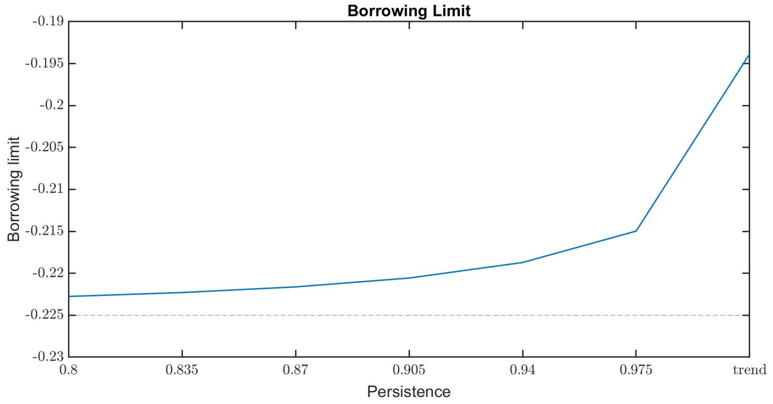
I simulate the model response to an MIT shock to income as described in Section 3.3 and compare the implications of different persistence of this shock.

Borrowing limit. Figure 2 shows the borrowing limit immediately after the unexpected drop in income. The horizontal line denotes the different values of ρ_Y for temporary shocks or indicates that the shock is hitting the trend. The red dashed line corresponds to the borrowing limit in the original steady-state. The graph illustrates that a negative shock causes the tightening of the borrowing constraint. Moreover, the more persistent output process corresponds to a stronger reduction in the credit supply. Notice that the response to the trend shock is approximately three times stronger than to a temporary shock of the highest persistence.

Intuitively, the tightness of the borrowing constraint depends on how much the value of repayment changes relative to the value of autarky. A lower expected path of output causes both values to go down. However, after a temporary shock, the value of repayment doesn't fall as much as the value of autarky, as the agent with access to financial markets can borrow and repay when income rebounds. In contrast, the trend shock causes a large drop in the value of repayment, as paying back the debt from the steady-state becomes harder under the permanently lower income. Thus, only low levels of debt are compatible with the no-default requirement.

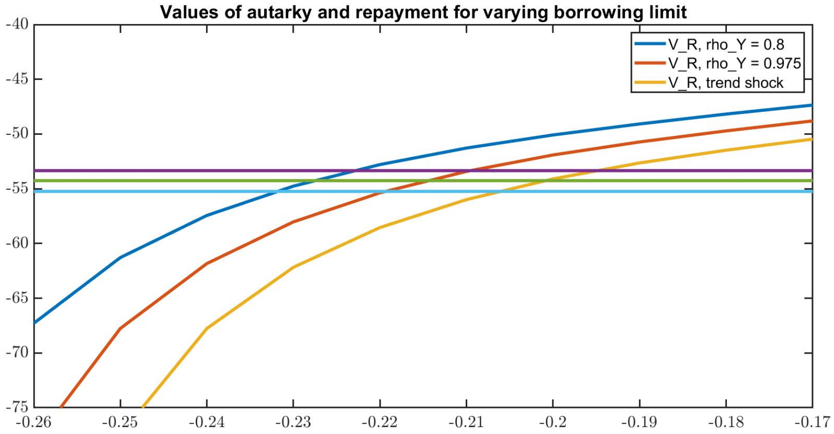
More formally, the determination of the debt limit for the trend shock and two temporary shocks with $\rho_Y = 0.8, 0.975$ is shown in Figure 3. There, the horizontal axis shows the debt limit immediately after the shock, at $t = 1$. It is determined by the intersection of the values of autarky (horizontal lines) and repayment (increasing curves) at $t = 2$. The values of autarky do not depend on the amount of debt due. The difference between the values of autarky under different shock processes is small and exclusively determined by the output trajectory. The values of repayment are increasing

Figure 2: Borrowing Limit Tightening for Output Shocks of Different Persistence



Notes: The blue solid line shows the tightness of the borrowing limit immediately after unexpected income shocks of different persistence, with numbers referring to the parameter ρ_Y. The red dotted line refers to the borrowing limit in the initial steady state.

Figure 3: Borrowing Limit Determination at $t = 1$



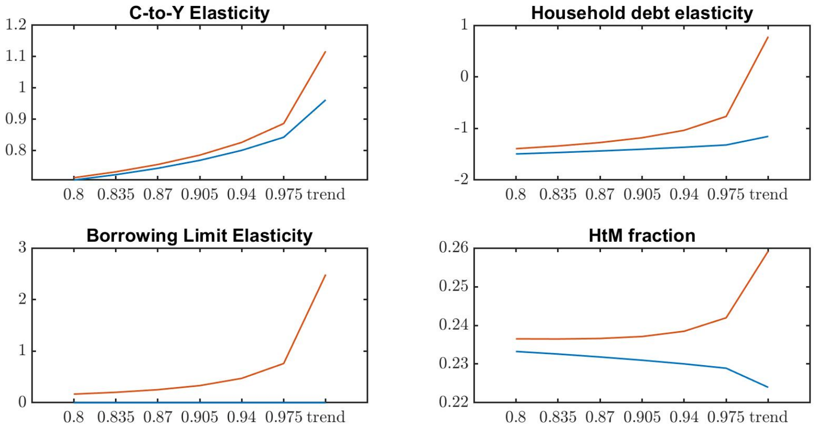
Notes: The horizontal axis depicts asset holdings and the vertical axis shows the value of autarky (horizontal lines) or repayment (increasing curves). The interception between the value of autarky and repayment determine the no-default borrowing limit. The values of repayment fall by more with higher persistence of the shock as it not only decreases permanent income, but also reduces the gains from consumption-smoothing.

in asset holdings and the distance between them depends not only on the output path but also on access to the financial market. Under temporary shock with $\rho_Y = 0.80$ (blue line), the household can benefit from borrowing today and repaying when the income returns to the steady state. Under a trend shock (yellow line), however, the household is not willing to borrow as it expects the income to remain low forever.

Debt deleveraging. The Figure 4 compares the benchmark model (blue line) and the model with no-default financial constraint (red line). Borrowing limit elasticity is defined as $\frac{\Delta \log \underline{a}_t}{\Delta \log Y_t}$, where $t = 1$, and denotes how much the limit tightens relative to the drop in output. For temporary shocks, this elasticity is lower than one. So, even despite the constraint tightening in the absolute terms, in the relative terms, $\underline{a}/Y$ loosens since the households are strongly interested in smoothing consumption. In contrast, after a trend shock, the elasticity reaches 2.5, which indicates tightening both in absolute and relative terms.

This difference leads to a sharp contrast in the model predictions for household debt. In the benchmark case, the households respond to the shock by increasing their borrowing. As the income

Figure 4: Elasticities of Borrowing Limit, Dent and Consumption



Notes: The graph compares the benchmark (blue) and no-default (red) model simulations for income shocks of different persistence, with numbers referring to the parameter ρ_Y. Elasticities are defined as $\frac{\Delta \log X_t}{\Delta \log Y_t}$, where $t = 1$ and X is either debt, consumption, or borrowing limit and denote how much these variables change relative to the output drop immediately after the shock hits. Hand-to-mouth share refers to the households with liquid assets less than two weeks of income.

goes down, the households try to use debt to support their consumption. Even in the trend shock case, there is some space for consumption-smoothing as low-income households expect their idiosyncratic income to improve eventually. With a transitory shock, the effect is even more pronounced due to the opportunity to smooth the aggregate shock as well. Under no-default constraint, the implications for temporary shocks are not that different due to the weak response of the borrowing limit. In contrast, the trend shock causes debt deleveraging driven by the sharp reduction in the credit supply. Therefore, only the combination of endogenous borrowing constraint and a trend output losses generate the debt deleveraging in this model, creating the association between financial tightness and permanent output losses.

Consumption and distributional implications. In addition, Figure 4 shows that the no-default borrowing constraint generates a stronger contraction in consumption and a higher share of hand-to-mouth households. Figure 5 depicts the distributional impact of the shock. It shows the elasticity of consumption to output for different income deciles. According to panel b), in the benchmark case, the elasticity is close to one for low-income, constraint agents. However, the unconstrained agents can smooth consumption, so in the case of a temporary shock, consumption doesn't decrease as much as output. In panel a), the behavior of high-income households is similar to the benchmark case. However, we can see large changes in consumption for low-income, hand-to-mouth agents, as their consumption falls not only because of lower income but also because of the restricted opportunity to borrow.

Also, notice that as the shock is unexpected, some households may choose to default. However, the fraction of such households is negligible at 0.1% for the endogenous borrowing limit and is even smaller for the benchmark model due to more opportunities to reroll the debt.

Amplification effect of the future constraints. Why the difference between borrowing limit response to trend and temporary shock is so large? The reason is the strong amplification of the current tightening by future binding constraints.

Figure 6 shows the model response under the assumption that the borrowing limit is only endogenous at $t = 1$, immediately after the shock. At $t \geq 2$, the constraint returns to the initial steady-state level (blue line). While the borrowing constraint still tightens more in response to a permanent shock, the magnitude of the change is insignificant, and the results remain close to the fixed constraint benchmark. Therefore, a major part of the borrowing limit elasticity results from

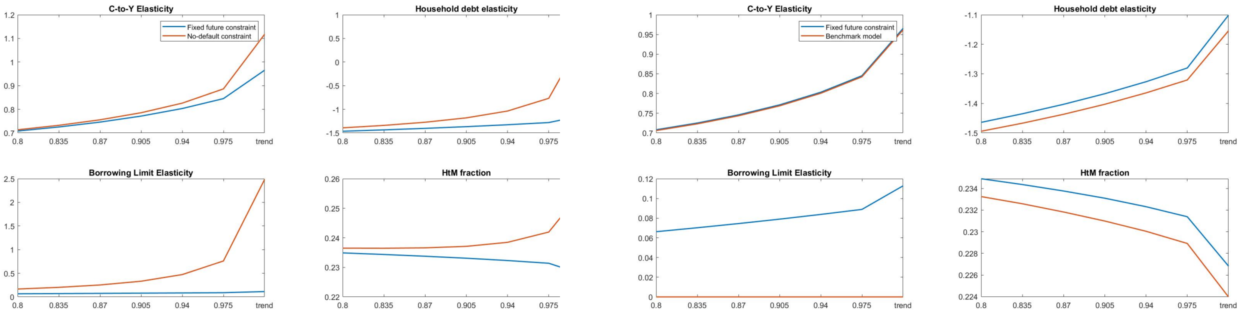
Figure 5: Consumption Elasticities by Income Decile

(a) Endogenous borrowing limit

(b) Benchmark

Notes: The figure shows elasticities of consumption to output in the cases of a trend shock and temporary shocks with $\rho_Y = 0.8, 0.975$, calculated as the elasticity of the average consumption by decile to the fall in output. Due to idiosyncratic component in income being multiplicative, the decline in income is uniform to all agents.

Figure 6: One-period Endogenous Constraint



(a) Endogenous borrowing limit

(b) Benchmark

Notes: The figure shows the model outcomes under assumption that the borrowing limit is only endogenous immediately after crisis and stays fixed at the original steady-state level afterwards (blue line). The results are compared with fully endogenous borrowing limit model and the benchmark model (red line in left and right panels respectively). The horizontal axis indicates the persistence of the shocks with numbers referring to aggregate persistence parameter ρ_Y.

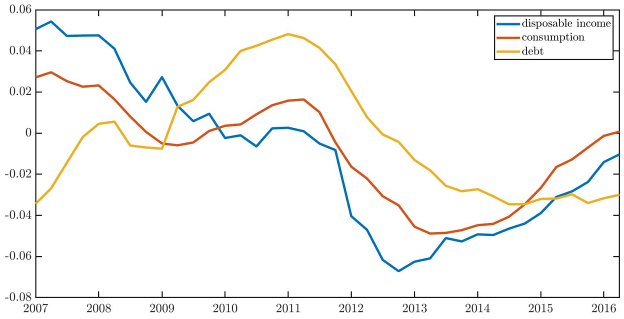
the future path of the borrowing constraint. Intuitively, it is the value of access to the credit market that determines the change in the no-default constraint. But the tightening of the constraint at $t \geq 2$ strongly reduces this value, making default a more attractive option. So, the current constraint must tighten as well.

This drives the distinction between trend and transitory shocks. Under the trend shock, the borrowing constraint remains tighter forever in the new steady state, so the value of access to the financial market falls due to the inability to borrow as much as in the initial steady state. Under temporary shock, in contrast, it eventually returns to the old steady state value. The difference in credit availability in the new steady state strongly affects the current borrowing limit due to its permanent nature.

4.2 Euro Crisis

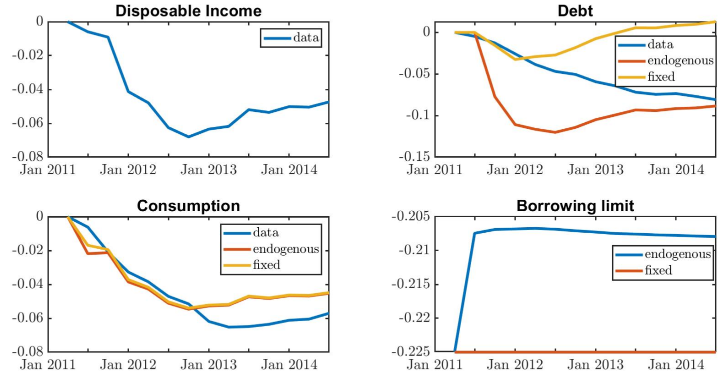
The previous section shows that, under no-default constraint, trend shocks to income can generate household debt deleveraging. The next question is whether this effect is economically significant and how it compares with the credit crunches observed in the data. This section compares the model with

Figure 7: Household Data for Euro Drisis, Italy



Notes: OECD data for Italy, per capita, seasonally adjusted, deviations from the linear trend

Figure 8: Data and Model Comparison, New Steady State at -4%



Notes: The figure shows how the model responds to the series of the income shocks estimated from data. The outcomes for income, consumption and debt are shown in the deviations from the initial steady state. The income is assumed to converge to -4% value.

the data on Italy during the Euro crisis (see Figure 7). I estimate the shocks to disposable income from 2011Q1 to 2014Q2, defining them as deviations from the linear trend, and feed them into the quantitative model[3]. I treat 2011Q1 as the steady state and assume that the crisis affected the trend income[4]. Moreover, I assume that in period 0 (2011Q1), the agents don't anticipate the crisis but learn the whole future path of income in period 1 (2011Q2).

Figure 8 shows how the data compares with the benchmark fixed-limit model and no-default limit model if the new steady state is -4% deviation from the old steady state. Under the no-default borrowing limit, the financial tightening occurs immediately after learning the news and doesn't fluctuate much in response to the current income. The steep response stems from the borrowing limit being mostly determined by the future path of the constraint, as shown in Section 4.1.

The differences in the credit supply generate diverging paths of the household debt. While in the data we can see gradual deleveraging, the benchmark model with the fixed constraint generates a small dip during the worst part of the recession, but afterward, the debt restores and even exceed the old steady-state value. In contrast, with the no-default constraint, we can see steep and persistent deleverage. Therefore, the endogenous constraint model manages to capture the medium-term dynamics of household debt, while the fixed constraint model cannot generate deleveraging[5]. In spite

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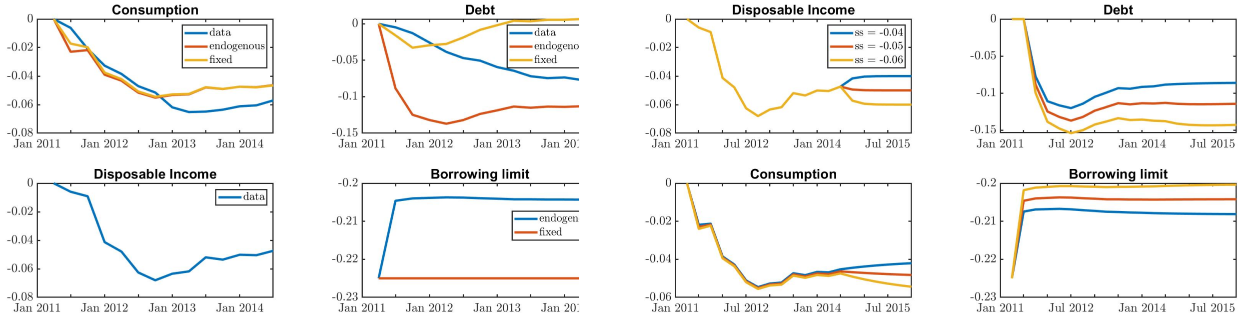
[3] Using annual data for consistency with calibration does not change the conclusions of this section.

[4] See Guntin et al. (2020) for the evidence that the Euro crisis can be treated as a shock to the trend.

[5] The steepness of the debt response in the model can be attributed to full adjustment of the expectation in the first period in the model and gradual change in expectations in the data. Alternatively, the debt dynamics in the data can be

Figure 9: Sensitivity to the New Steady State Assumption



(a) Data and model comparison, new steady state at -5%

(b) Endogenous constraint model with different steady states

Notes: The left panel shows how the model responds to the series of the income shocks estimated from data. The outcomes for income, consumption and debt are shown in the deviations from the initial steady state. The income is assumed to converge to -5% value. The left panel compares the model output under -4%, -5% and -6% steady-state assumptions using a longer time horizon.

of the new steady state being higher than the data suggests, the deleverage under the no-default constraint model slightly exceeds the one in the data.

Robustness. The results in this section are sensitive to the assumption on the new steady state. The left panel of Figure 9 repeats the exercise for the new steady state of -5%, which is closer to the end points in the data. The medium-run values for debt become 3 percentage points lower than under the -4% assumption and clearly overestimate the credit crunch observed in the data. The right panel of Figure 9 illustrates the dynamics of debt and consumption with a longer horizon, showing how the assumption on the new steady state of income has important implications for the dynamics of debt.

The mechanism behind this sensitivity is the same as the one behind drastic differences in debt response to transitory and permanent shocks. A lower steady-state income is associated with a permanently tighter financial constraint, which strongly reduces the value of access to the credit market and causes the no-default constraint to become more binding. The resulting restriction in credit supply determines the magnitude of the debt deleveraging.

Distributional impact. While the dynamics of the aggregate consumption is similar under endogenous and exogenous borrowing limit, the two models have different predictions for the distributional impact of the shock.

Figure 10 depicts the consumption elasticity to output for different income deciles in the first period after the crisis starts. As the output fall is gradual, the households mostly react to the expectations, which results in the elasticity for unconstrained households being larger than one in both models. The response of low-income households, however, is drastically different. Under the no-default limit, the inability to borrow strongly reduces the consumption of low-income hand-to-mouth households. In contrast, under the fixed borrowing limit, low-income households exhibit one-to-one contraction in consumption, which is a weaker response than one of the unconstrained households. Even with lower optimal consumption, these households cannot afford it and remain hand-to-mouth, consuming all the available resources.

The empirical results on cross-sectional differences in consumption adjustment provide mixed support to this result. Guntin et al. (2020) show that the consumption adjustment is relatively flat across income deciles and low-income households tend to adjust less, similar to the fixed constraint model. However, the model does not feature the wealthy hand-to-mouth households, which may

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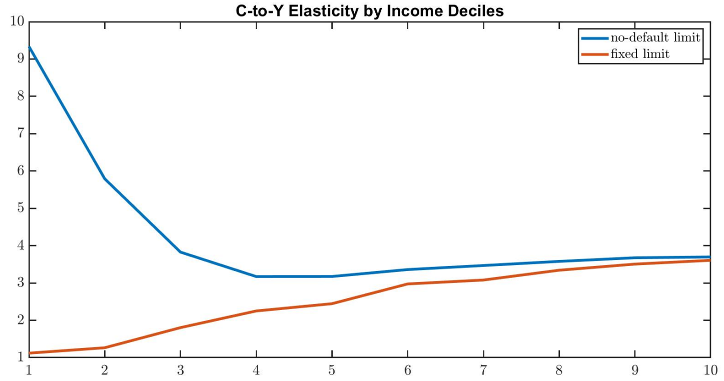
smoother due to multi-period debt.

Figure 10: Consumption Elasticities by Income Decile



Notes: The figure shows elasticities of consumption to output by income decile in the first period for the models with endogenous (blue) and exogenous (red) financial constraints. The elasticities are calculated as the elasticity of the average consumption by decile to the fall in output. Due to idiosyncratic component in income being multiplicative, the decline in income is uniform to all agents.

exacerbate the difference between the elasticities for low- and high-income households. Moreover, low-income households can also be interpreted as the ones holding the debt. Kovacs et al. (2018) show on the UK data that highly indebted households cut their consumption expenditure by more. They cite a survey explaining it with tighter credit constraints and concerns about the inability to repay the debt in the future. Andersen et al. (2014) report similar results for Denmark. That can be interpreted as evidence supporting the no-default constraint model.

4.3 Discussion

As shown above, under the assumptions of the model, the association between financial tightness and permanent output loss can be generated with the no-default borrowing limit. It tightens steeply in response to a trend shock to income, causing a credit crunch and debt deleveraging. This effect is specific to the combination of the endogenous constraint and a trend shock and is does not occur with the fixed borrowing limit or transitory shocks. Below, I discuss how the results depend on the model assumptions and possible approaches to studying their empirical relevance.

Borrowing limit dependence on the new steady state. As shown in the previous section, nearly all of the credit tightening comes not from the fall in income itself but from the lower access to credit in the future, which makes default a more viable alternative to repaying the debt. The magnitude of this effect may depend on the assumption of no redemption after a default. If the model allows the household in autarky to reenter the financial market eventually, the credit crunch would still depend on the tightness of the constraint in the nearest future but not directly take into account the borrowing limit in the new steady-state. That may reduce the difference between trend and transitory shocks. In future work, one direction should be to explore if the findings are robust to the possibility of redemption and if such a model still generates the debt deleverage after trend shocks.

Overestimating the debt deleveraging. Other assumptions can also matter for the strength of the deleveraging. First, the model assumes that the default cost is fixed in the utility units. If instead, it is denominated in the units of consumption, the utility equivalent would increase during recessions, making defaults less attractive and the borrowing limit less elastic. Second, in the model, the financial sector reacts to an increase in the probability of default by restricting the supply but keeping the cost of borrowing fixed. In reality, one may expect to see a combination of the two with some default allowed in equilibrium at the cost of higher interest rates. In a model where the two mechanisms coexist, cheap credit would be rationed, but the credit crunch would be milder because of the availability of the more expensive credit.

The role of heterogeneity. While, as the analytical example shows, the result of borrowing limit tightening would hold even in the representative agent case, the MIT shock experiment requires heterogeneity. In a RA model where the borrowing limit binds in the steady state, the MIT shock implies the default of the representative agent. Then, the deleveraging happens not due to the credit supply contraction but due to the loss of access to the financial market. If the constraint is not binding, it is the distance from the borrowing constraint that defines if the deleveraging happens. Furthermore, in the RA model, the borrowing limit itself would be less elastic to changes in income since, in the HA case, the financial market provides insurance not only against the aggregate shocks but also against the idiosyncratic ones.

Empirical evidence. While the model successfully generates financial tightening in response to a trend income shock, the question remains whether the channel is present in the economy. The challenge is to tell the endogenous contraction in credit supply apart from a financial shock causing persistent output losses. For this purpose, it is necessary to establish which predictions of the two models differ. One type of evidence can be if the change in the credit supply is permanent. In most models, financial shocks are transitory, so the contraction in credit supply should gradually dissipate. On the other hand, the endogenous response to a trend output shock would result in a tighter credit supply in the new steady state.

Another approach is to try and document the empirical relevance of the no-default constraint. One way to look for the evidence of the no-default limit is to explore how credit limits respond to non-financial crises (for example, Covid or political crises) and whether they are sensitive to expectations. If they contract after a shock originating outside the financial system or deterioration in economic prospects, that can be evidence of forward-looking endogenous constraint. Another way is to study individual-level data. Under the no-default limit, credit availability for households employed in the industries least affected by a recession should exceed the one for households in the most affected industries due to the differences in their expected income. Moreover, under the no-default constraint hypothesis, consumption adjustment of hand-to-mouth households should exceed one of the unconstrained households as shown in the previous section.

5 Policy Implications

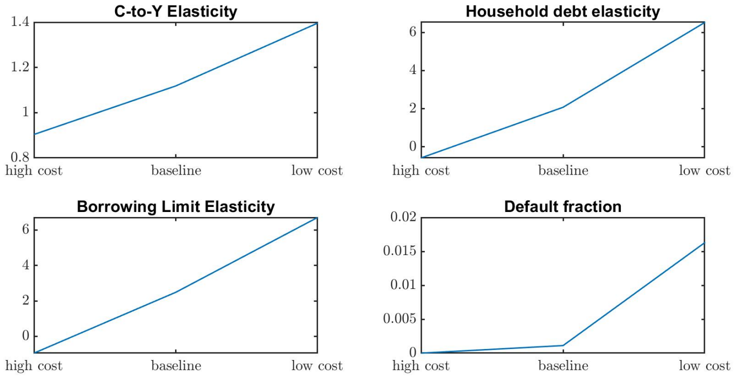
In this section, I use the quantitative model to consider a government permanently lowering the penalty for default as a measure to improve the consumption of low-income households during the crisis. I show that with the endogenous borrowing limit, this policy would stimulate the consumption of households who choose to default but at the same time exacerbate financial tightening, hurting other households. In aggregate, this policy lowers consumption.

A government wishes to support the consumption of low-income households during the crisis and implements a policy changing the cost of default. Immediately after the recession starts, the government either lowers or increases the utility cost of default by 20% of the baseline value. One can interpret setting the lower cost as simplifying the bankruptcy procedures or adding protections in the case of bankruptcy. I assume the change to be permanent, as in a case when a recently introduced regulation is hard to undo. In the case of default, the household repays the penalty in units of current consumption in the same period.

Under the fixed borrowing constraint, this policy would unambiguously support consumption. A household chooses to default if the immediate gains of increasing consumption are high enough to compensate for entering autarky. However, with endogenous borrowing constraint, this policy creates a trade-off. While still raising the current consumption of the defaulting households, it also has implications for the borrowing limit. As a low default cost makes repaying the debt next period less attractive, the financial constraint tightens. That may affect households who are unwilling to default.

To study which effect prevails, I conduct a policy experiment, changing the default cost after a

Figure 11: Aggregate Implications of Changes in Default Cost



Notes: The figure compares the aggregate responses of the baseline model and the models where on the onset of the recession, the cost of default is set to be 20% higher or lower. The fraction of defaults refers to their share among all households, not only the borrowers. The recession is modelled as a 5% trend shock to output.

permanent 5% output drop. Figure 11 shows how the policy intervention impacts the aggregate variables. First, under the low penalty, the fraction of defaults increases more than ten times and reaches 1.5% of all households, while under the baseline parameters, this share was close to 0.1%. Second, the borrowing limit reacts strongly to the changes in policy. The constraint becomes more than two times more elastic if the cost is low, while the penalty increase makes the borrowing limit loosen compared to the original steady state. The response of the borrowing limit has strong implications for debt and consumption: under the low (high) penalty, they fall more (less) than in the baseline. So, in aggregate, a cost decrease is not effective in supporting consumption.

The differences across types of households help to explain this result. The left panel of Figure 12 shows the elasticities of consumption for all income deciles, while the right panel zooms in on the left tail of the distribution and depicts the more detailed information on the 5% of households with the lowest income. The right panel depicts that for the first 1.5% households, the low cost of default helps to stimulate consumption, as these households choose to default. However, even among the 5% of the lowest income households, most agents lose from this policy. They repay the debt but are restricted in getting credit. The left panel shows that, under the policy, the lowest decile has lower consumption on average. Moreover, consumption falls even for higher income deciles, possibly due to a precautionary savings increase. The policy's failure to reach its goal can be explained by the low share of default in this model.

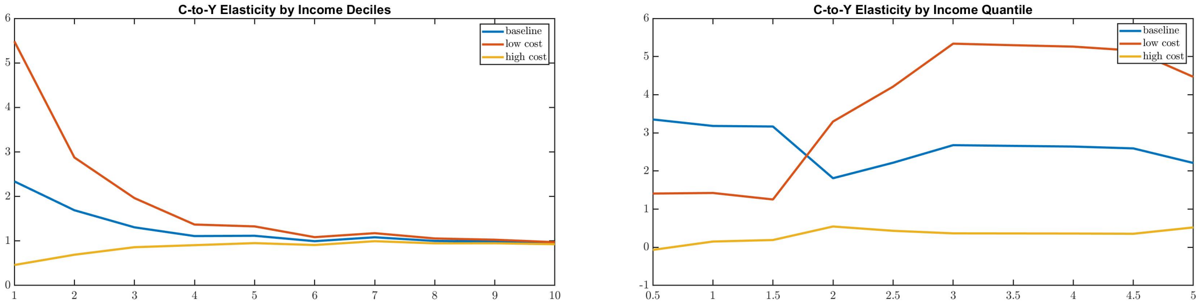
Another result of this exercise is that increasing the default cost raises consumption for all households due to the looser borrowing constraint. In interpreting this result, one has to take into account that in this model, all the defaults are strategic. Every household can repay the debt if they prefer it to autarky. If some households have to default due to insufficient liquidity, a higher default penalty would unambiguously hurt these households.

In appendix B, I show that the main conclusions don't change if the policy is reverted after several periods. If the default cost lowers temporarily, the borrowing limit does not tighten as much as under the permanent policy. But the share of defaults also decreases since the households expect to benefit more from the credit market and are less willing to enter autarky. The policy still reduces consumption even for the lowest income quantiles.

6 Conclusion

This paper suggests a new channel connecting financial crises and slow recovery by taking the output process to be exogenous and endogenizing the credit constraints. Under a no-default borrowing limit,

Figure 12: Distributional Implications of Changes in Default Cost



(a) Consumption Elasticities by Income Decile

(b) Consumption Elasticities for Low Income Quantiles

Notes: The figure shows elasticities of consumption to output by income quantiles in the first period for the baseline model and the models where on the onset of the recession, the cost of default is set to be 20% higher or lower. The elasticities are calculated as the elasticity of the average consumption by decile to the fall in output.

trend shocks generate a much stronger credit tightening than transitory shocks. When explored in a heterogeneous agent Aiyagari-type model, it generates qualitatively different predictions for the response of household debt to transitory and trend shocks: after a trend shock, the credit supply falls enough to cause household debt deleveraging. The debt contraction is sizable and overestimates the one observed in the data.

There are two main directions for further work. The first is ensuring the model is robust to its assumptions. In particular, the assumption of no regaining access to the credit market after default can result in a larger difference between trend and transitory shock. It is worth checking if, in a model with a possibility of redemption, the trend shock still reduces the credit supply enough to cause deleverage. Second, empirical evidence is needed to support the relevance of the no-default constraint and plausibility of the suggested channel. It may base on cross-sectional differences in consumption adjustment and medium-term behavior of credit supply after a crisis.

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Appendix A: Proofs for analytical example

6.1 Low income results in tigher constraint

Given $\{y\}_1^\infty$, if $y_0^1 < y_0^2$, then $\underline{a}(y_0^1) > \underline{a}(y_0^2)$. Low income y_0 is associated with the tighter borrowing limit $\underline{a}(y_0)$.

Proof. The proof is a modified version of the proof in Arellano (2008).

Step 1: Show that default is only possible if there is no rollover opportunity. If $a < \underline{a}(y)$, then cannot be that $(1+r)a - a' > 0$.

Suppose there exists a' s.t. rollover is possible, but default is preferred. Denote $\hat{a}$ as the optimal choice of borrowing under repayment. Then,

$$V_R(y,a) = u(y + a(1+r) - \hat{a}) + \beta \mathbb{E} V_O(\hat{a}, y') < V_A(y),$$

where $V_O(\cdot)$ is the value of choosing between default and repayment at the beginning of the period. But notice that

$$u(y + a(1+r) - a') + \beta \mathbb{E} V_O(\hat{a}, y') \geq u(y) + \beta \mathbb{E} V_A(y'),$$

The first term is larger as $a' \geq a(1+r)$, and $V_O(\hat{a}, y')$ implies that the option of default is available, so it is at least as large as $V_A(y')$. But then $\hat{a}$ cannot be the optimal choice.

Step 2. For all $y_1 \leq y_2$, if $V_R(y_2, a) \leq V_A(y_2)$, then $V_R(y_1, a) \leq V_A(y_1)$. If the agent defaults in the state y_2, then the agent also defaults in the lower state y_1.

Write $V_R(y_2, a) \leq V_A(y_2)$ explicitly as

$$u(y_2) + \beta \mathbb{E} V_A(y') > u(y_2 + a(1+r) - a') + \beta \mathbb{E} V_O(a', y')$$

To get the results, it suffices to show that the difference of repayment and default values would be lower for y_1 than for y_2. After rearrangement, that would give

$$u(y_2 + a(1+r) - a'') + \beta \mathbb{E} V_O(a'', y') - u(y_1 + a(1+r) - a') - \beta \mathbb{E} V_O(a', y') > u(y_2) + \beta \mathbb{E} V_A(y') - u(y_1) - \beta \mathbb{E} V_A(y'),$$

where a' is the optimal choice of assets under y_1 and a'' is the optimal choice of assets under y_2. Due to the fact that the income process is either deterministic or iid, the values of autarky on the right hand side cancel out. From a'' being the optimal choice, also know that

$$u(y_2 + a(1+r) - a'') + \beta \mathbb{E} V_O(a'', y') \geq u(y_2 + a(1+r) - a') + \beta \mathbb{E} V_O(a', y')$$

So, the claim would hold if

$$u(y_2 + a(1+r) - a') + \beta \mathbb{E} V_O(a', y') - u(y_1 + a(1+r) - a') - \beta \mathbb{E} V_O(a', y') > u(y_2) - u(y_1)$$

$$u(y_2 + a(1+r) - a') - u(y_1 + a(1+r) - a') > u(y_2) - u(y_1)$$

From step 1 and (y_2, a) being a default state, we know that $a(1+r) - a' < 0$. Then the inequality holds from the utility function being increasing and strictly concave.

Step 3. If $y_1 \leq y_2$, then $\underline{a}(y_1) \geq \underline{a}(y_2)$. Low income y_0 is associated with the tighter borrowing limit $\underline{a}(y_0)$.

Imagine $\underline{a}(y_1) < \underline{a}(y_2)$. Then, by definition, $V_R(y_2, \underline{a}(y_1)) < V_A(y_2)$. But we know that $V_R(y_1, \underline{a}(y_1)) = V_A(y_1)$. It contradicts the step 2. □

6.2 Higher persistence of a shock results in a tighter constriant

In the case of an AR(1) drop in income, for any $\rho \in (0,1)$, $\underline{a} < 0$. Furthermore, if $\rho_1 > \rho_2$, then $a_1 > \underline{a}_2$, so that the borrowing limit tightens with persistence.

Proof. The values of default and repayment are

$$V_A = u(y_H - \epsilon) + \beta u(y_H - \rho\epsilon) + \beta^2 u(y_H - \rho^2\epsilon) + ...$$

$$V_R = \frac{1}{1-\beta} u(\frac{1-\beta}{\beta} a + y_H - \frac{1-\beta}{1-\beta\rho}\epsilon)$$

As the value of autarky is difficult to simplify, I prove the claim by induction.

Base of two periods. Assume that the agent lives only for two periods but only have to repay the interest rate ra on the debt in both periods. Want to show that the borrowing limit is negative and tightens with persistence. For the first, it suffices to show that the left hand side of

$$u^{-1}\left[\frac{1}{1+\beta}\left(u(y_H - \epsilon) + \beta u(y_H - \rho\epsilon)\right)\right] - y_H + \frac{1+\beta\rho}{1+\beta}\epsilon = \frac{1-\beta}{\beta} a$$

is negative. That condition can be rewritten as

$$\frac{1}{1+\beta}\left(u(y_H - \epsilon) + \beta u(y_H - \rho\epsilon)\right) \leq u\left(y_H - \frac{1+\beta\rho}{1+\beta}\epsilon\right)$$

Notice that the left hand side is the weighted average of $u(y_H - \epsilon)$ and $u(y_H - \rho\epsilon)$, while the right hand side is the function $u(\cdot)$ of the weighted average of $y_H - \epsilon$ and $y_H - \rho\epsilon$. The inequailty holds due to the concavity of $u(\cdot)$.

For the second, need to show that the difference between the two sides is increasing in ρ. That gives the condition

$$u'(y_H - \rho\epsilon) - u'(y_H - \frac{1+\beta\rho}{1+\beta}\epsilon) \leq 0$$

As $\rho \leq \frac{1+\beta\rho}{1+\beta}$, this condition holds. So, higher persistence means tighter constraint.

Step of induction. Assume that for $n-1$ we know that

$$\frac{1}{\Sigma}\left(u(y_H - \epsilon) + ... + \beta^{n-1}u(y_H - \rho^{n-1}\epsilon)\right) - u\left(y_H - \frac{\sum_{i=0}^{n-1}\beta^i\rho^i}{\Sigma}\epsilon\right)$$

is negative and increasing in ρ, where $\Sigma = \sum_{i=0}^{n-1}\beta^i$. Want to show the same for n terms. Denote the first term of this expression as LHS and the second as RHS. Then, the analogous expression for n terms can be written as

$$\frac{\Sigma}{\Sigma+\beta^n} LHS + \frac{\beta^n}{\Sigma+\beta^n} u(y_H - \rho^n\epsilon) - u\left(\frac{\Sigma}{\Sigma+\beta^n} u^{-1}(RHS) + \frac{\beta^n}{\Sigma+\beta^n}(y_H - \rho^n\epsilon)\right)$$

Replace LHS with RHS. According to the $n-1$ step, the difference between LHS and RHS is increasing in ρ, so if we show that after the substitution the expression is also increasing in ρ, it would be also true for the original expression.

Next, define the following:

$$\epsilon' = \epsilon \frac{\sum_{i=0}^{n-1} \beta^i \rho^i}{\Sigma}$$

$$\rho^n = \rho' \frac{\sum_{i=0}^{n-1} \beta^i \rho^i}{\Sigma}$$

$$\frac{\Sigma}{\Sigma + \beta^n} = \frac{1}{1+\beta'}$$

Notice that

$$RHS = u\left(y_H - \frac{\sum_{i=0}^{n-1} \beta^i \rho^i}{\Sigma} \epsilon\right) = u(y - \epsilon')$$

Then, do the substitutions:

$$\frac{1}{1+\beta'} u(y - \epsilon') + \frac{\beta'}{1+\beta'} u(y_H - \rho'\epsilon') - u\left(\frac{1}{1+\beta'}(y - \epsilon') + \frac{\beta'}{1+\beta'}(y_H - \rho'\epsilon')\right)$$

Notice that it is identical to the base of induction. As ρ' is increasing in ρ, this expression is also increasing in ρ.

The argument for the limit. As $n \to \infty$, both sides converge to a number due to exponentially lower weights of the new terms and the terms themselves converging to y_H. Want to show that in the limit the sign is not reversed. Assume it is, and the difference between the limits is ε. Then, there must be a finite n such that the distance from the limit for both terms is less than $\frac{\varepsilon}{2}$ and $LHS > RHS$, which contradicts the previous step. Moreover, due to concavity, equality can hold only when all the terms are equal, or in the case of the permanent output drop. Therefore, the borrowing limit must be negative.

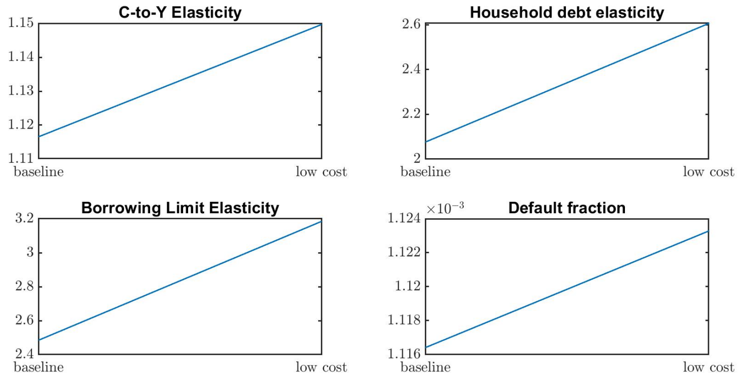
As for the second part of the proposition claiming that the borrowing limit is increasing in ρ, a similar argument can be used to compare the income processes with two different persistence and show the sign of the inequality between the borrowing constraints cannot be reversed in the limit.

□

Appendix B: Temporary Reduction in Default Cost

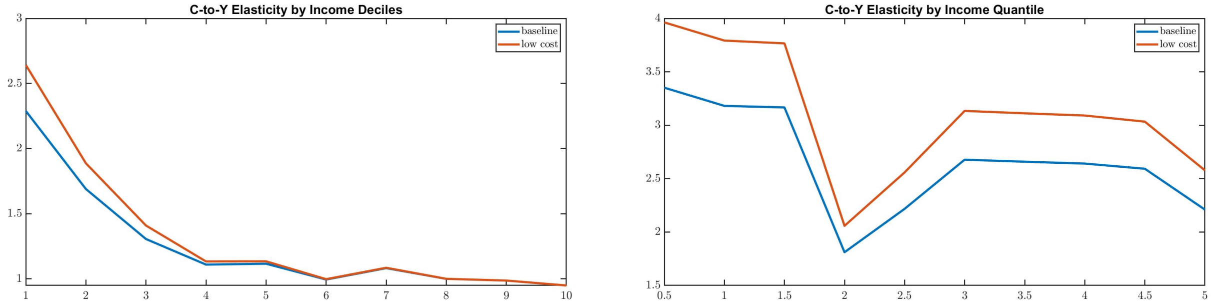
Here, I check the robustness of the policy analysis by assuming that the cost of default is lowered immediately after a crisis by 20% but is reverted to the initial value after ten periods. I find that the tightening of the financial constraint is much milder than under the permanent policy change. At the same time, the share of defaults doesn't increase much compared to the no policy intervention case, as default becomes less attractive when the future borrowing constraint doesn't tighten as much. The policy still fails to stimulate the consumption of low-income households. The Figures below illustrate these conclusions.

Figure 13: Aggregate Implications of Changes in Default Cost



Notes: The figure compares the aggregate responses of the baseline model and the models where on the onset of the recession, the cost of default is set to be 20% lower. The fraction of defaults refers to their share among all households, not only the borrowers. The recession is modelled as a 5% trend shock to output.

Figure 14: Distributional Implications of Changes in Default Cost



(a) Consumption Elasticities by Income Decile (b) Consumption Elasticities for Low Income Quantiles

Notes: The figure shows elasticities of consumption to output by income quantiles in the first period for the baseline model and the models where on the onset of the recession, the cost of default is set to be 20% lower. The elasticities are calculated as the elasticity of the average consumption by decile to the fall in output.